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# BRITTLE SOLID UNDER COMPRESSION. PART II: THE PROBLEM OF MACRO TO MICRO LINKAGE

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**Abstract**—The macrodescription of the nonlinear behavior of a heterogen under compression, as given by the central function: $\sigma = \varepsilon EG$, employs the Gaussian term $G$, which expresses the stochastic character of the heterogen's atrophy through microcracking and comprises two critical parameters: the limiting strain of elasticity (atrophy threshold)—$\varepsilon_a$, and the scattering factor—$d$. It does not, however, involve parameters of the microcracking mechanisms. As for the models of local lateral tension and microrupture, they are based on the gradients in Poisson's ratio and elastic moduli of the heterogen's components and are not related to the parameters of the heterogen in macro.

Linkage of the micro and macro models was effected by generalization of the gradient micromechanisms. It showed that the strength of a brittle solid in compression is a function of its resistance to microrupture and of the gradient factor. Despite the fact that *longitudinal compression* creates the gradient strains, it is shown that the two critical macroparameters, $\varepsilon_a$ and $d$, are affected (in a probabilistic way) by two *critical lateral* strains: the minimal limiting strain of microrupture—$\varepsilon_a^R$ and the strain of the mode (the maximum) of the *pdf* of the heterogen's resistance to microrupture—$\varepsilon_M^R$.

To check the obtained models, the linkage between the strength of concrete in tension and compression was analyzed. It was found that their ratio reflects the gradient factor in the heterogen. The decrease in Poisson gradient with increasing strength can explain the faster increase in the compressive strength versus the tensile one. The obtained results confirm an old idea that the properties of a heterogen in tension represent its fundamental characteristics.

In the light of the obtained models the failure of concrete in the Brazilian test seems not to be another type of tensile failure, but a distinct kind of degeneration of the heterogen due to gradient lateral strains induced through the strips under compression. © 1997 Elsevier Science Ltd.

## NOMENCLATURE

| | |
|---|---|
| Heterogen | brittle heterogeneous solid |
| SSc | the curve of stress-strain relationship |
| atrophy | degeneration of heterogen due to microcracking |
| *pdf* | probability density function |
| $E$ | elastic modulus |
| $\nu$ | Poisson's ratio |
| $\delta$ | gradient factor |
| $\varepsilon, \varepsilon_2$ | longitudinal and lateral strain, induced by loading |
| $\epsilon, \epsilon^*$ | transverse strains and their gradient, respectively |
| $\sigma$ | stress |
| $G$ | Gaussian |

## 1. MICRORUPTURE AND MACROSTRENGTH

In the first part of this paper a description of some mechanisms of gradient strains in a brittle solid (heterogen) was given. The most general are the mechanisms which, due to differences in Poisson's ratio between the brittle components, *internally* induce gradient strains of transverse tension *under compression*. These very local gradient strains are the origin of the microcracking and incremental degeneration of the brittle solid under load. By contrast, the longitudinal—$\varepsilon_1$, and lateral—$\varepsilon_2$ strains of *external* compression, which are the only origin of the stress-strain state of the heterogen, affect it equally, in macro, throughout its bulk.

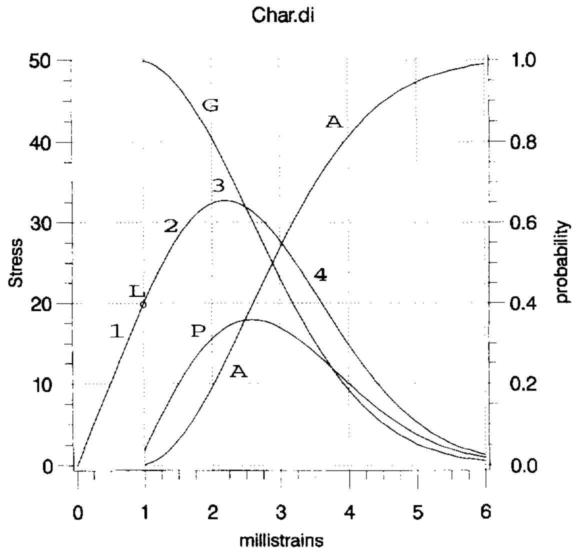


Fig. 1. Characteristic diagram (Char.di) of brittle solid under compression.

In Blechman (1988, 1992) the nonlinear behavior of concrete and its strength was described in terms of stochastic atrophy, which reflects the microcracking process of degeneration in macro only.

However, to understand the fundamentals of the behavior and strength of a heterogen under load, it is necessary to link the parameters of the macro-models with those of the microrupture mechanisms in it.

## 2. MACROLEVEL

### 2.1. *Central function*

According to Blechman (1992), a family of curves reflecting the behavior of a heterogen under load is given in a descriptive (characteristical) diagram (Fig. 1). The stress-strain relationship is represented by curve 1-2-3. The description of the linear part 1 is

$$\sigma = \varepsilon E.$$

The non-linear part 2, located between the atrophy threshold (the limit of linearity) $\varepsilon_a$ and the peak point $\varepsilon_p$, is expressed by the following central function:

$$\sigma = \varepsilon E G, \tag{1}$$

where $\sigma$ = stress, $\varepsilon$ = longitudinal strain, in the domain $\varepsilon_a < \varepsilon < \varepsilon_p$, $E$ = elastic modulus, and $G$ is the Gaussian, (curve $G$ in Fig. 1), which expresses the probability of survival of the heterogen as a function of the strain-distance $\{\varepsilon - \varepsilon_a\}$ from the atrophy threshold in the following form:

$$G = \exp\left(-0.5\frac{(\varepsilon-\varepsilon_a)^2}{d^2}\right), \tag{2}$$

where $d$ is the scattering factor. As shown in Blechman (1992)

$$d^2 = \varepsilon_p(\varepsilon_p - \varepsilon_a). \tag{3}$$

Part 3 of the curve 1-2-3 is its peak domain and part 4 is the descending branch.

2.2. *Strength*

The equation of heterogen's strength $\sigma_o$ simply follows from eqn (1), when $\varepsilon = \varepsilon_p$ is the peak point of the SSc

$$\sigma_o = \varepsilon_p E G_p, \tag{4}$$

where $G_p$ is the value of Gaussian at the peak point

$$G_p = \exp\left(-0.5\left(1-\frac{\varepsilon_a}{\varepsilon_p}\right)\right). \tag{5a}$$

Or, using (3),

$$G_p = \exp\left(-0.5\frac{d^2}{\varepsilon_p^2}\right). \tag{5b}$$

2.3. *Atrophy*

The cumulative atrophy (degeneration) of a heterogen $A$ during the stage of micro-cracking $\varepsilon_a < \varepsilon < \varepsilon_p$, (curve A in Fig. 1) is given by the expression

$$A = 1 - G. \tag{6a}$$

By definition the atrophy is a macro-characteristic of a heterogen as a whole, where the contribution of a single microcrack cannot be reflected.

The limiting value of cumulative atrophy $A_p$ which corresponds to the peak point of the heterogen's strength, is found when (5a) is substituted in eqn (6a):

$$A_p = 1 - \exp\left(-0.5\left(1-\frac{\varepsilon_a}{\varepsilon_p}\right)\right). \tag{6b}$$

The probability density function (*pdf*) of the heterogen's atrophy in macro—$P_A$, can be obtained by differentiating the cumulative function (6a):

$$\mathrm{d}A = P_A\,\mathrm{d}\varepsilon. \tag{7a}$$

Then

$$P_A = \mathrm{d}A/\mathrm{d}\varepsilon. \tag{7b}$$

Differentiating eqn (6a), when $G$ is taken from eqn (2) we obtain

$$P_A = \frac{\varepsilon-\varepsilon_a}{d^2}G. \tag{7c}$$

The expression for $P_A$-*pdf* has the form of the Rayleigh distribution (curve $P$ in Fig. 1). Its important feature is the linkage between three parameters: $\varepsilon_M$—the strain of the mode (maximum) of this *pdf*, the $\varepsilon_a$—its initial point, and $d$—the scattering factor

$$\varepsilon_M = \varepsilon_a + d. \tag{8}$$

2.4. *Atrophy threshold*

The atrophy threshold $\varepsilon_a$ is the point where the stress-strain curve deviates from the usual initial straight line due to onset of the microcracking process. Three special cases should be noted:

(1) For low-strength concrete and some soft rock materials there is no initial straight line. The stress-strain curve is non-linear from the beginning, and $\varepsilon_a = 0$.

(2) In concrete and some rock materials the stress-strain curve has a concavity at its beginning, with a straight line after it.

(3) Weak materials may have, instead of a straight line after the concavity, an inflection point only.

In all cases the atrophy threshold is readily found from the experimental $\sigma-\varepsilon$ graph or by AE (acoustic emission).

### 2.5. *Parameters of stress-strain curve*

According to Blechman (1988, 1992), the macrodescription of the stress-strain relationship in a heterogen is based on three parameters: $\{E, \varepsilon_a, d\}$. The elastic modulus $E$ is a constant of a continuum, known from experiments, and $\{\varepsilon_a, d\}$ are the parameters of the atrophy (degeneration) of the heterogen under loading, which are easily derived from experimental stress-strain curve.

In practice, beside $E$, only $\varepsilon_a$ and $\varepsilon_p$ are measured in experiments, but since the four parameters of interest: $\{\varepsilon_a, \varepsilon_p, d, \varepsilon_M\}$ are linked by eqns (3) and (8), we can find $\{d, \varepsilon_M\}$ by the same means.

As shown in Blechman (1988), the intrinsic parameters of SSc are $E$, $\varepsilon_a$ and $d$ or $\varepsilon_M$, while $\varepsilon_p$ can be expressed by $\varepsilon_a$ and $d$, as follows

$$\varepsilon_p = 0.5\varepsilon_a + \sqrt{(0.5\varepsilon_a)^2 + d^2}. \tag{9}$$

## 3. MICROLEVEL

As shown in Part 1, the fundamental parameters of a brittle solid at the level of the micromechanisms are the gradient of transverse strains and the local resistance to microrupture.

### 3.1. *Microrupture—limiting strain*

The highly localized and highly restricted internal tension induced by gradient strains can be defined as *restricted microtension*. The resistance of the heterogen to this microtension is expressed by the limiting (local) strain of microstructure $\varepsilon^R$ as follows

$$\epsilon^R = \min\{\epsilon_t, \epsilon_{int}\} - \epsilon_r - \epsilon_{pp}. \tag{10}$$

Here, the first term is the lower of the following two: $\epsilon_t$ = the limiting microstrain of the component's resistance to microrupture, and $\epsilon_{int}$ = the microstrain of the resistance to tension of the components' interfaces. The other two terms are: $\epsilon_r$ = the strain of residual tension in the heterogen and $\epsilon_{pp}$ = the lateral strain induced in the heterogen by pore pressure of liquid or gas.

### 3.2. *Gradient factors*

Following Part 1, some gradient mechanisms can induce the gradient factor $\delta$ in a heterogen. The first represents the Poisson gradient between aggregate and matrix, as per eqn (16) in Part 1 as follows

$$\delta_v = (k_a v_a - k_m v_m)\rho_m. \tag{11a}$$

Here, $v_a$, $v_m$ = Poisson's ratio for aggregate and matrix, respectively, and $\rho_m$ = aggregate/

matrix stiffness ratio in longitudinal cross-section. There is also an alternative expression for the Poisson gradient, given by eqn (18) in Part 1:

$$\delta_v = v_o - v_m, \tag{11b}$$

where $v_o$ is the average value of Poisson's ratio of the heterogen.

The second gradient factor is that of the thrust in a matrogen:

$$\delta_E = k\frac{E_a}{E_m} - 1. \tag{11c}$$

Here, $k$ = a coefficient, $E_a$, $E_m$ = elastic moduli of aggregate and matrix, respectively.

The third gradient factor is that of a crystalon, where due to differences in Poisson's ratio along the main axes of the crystals, laterally tensioned crystals—acrons—appear and are affected by the following gradient, (eqn (26) in Part 1)

$$\delta_{acr} = f_a\delta_a - \omega\delta_c. \tag{11d}$$

When the axis of maximum Poisson's ratio of the crystal, $v_{pst}$ is oriented laterally, the crystals behave as "pistons", with gradient factor $\delta_{pst}$

$$\delta_{pst} = v_{pst} - v_o, \tag{11e}$$

and for the state of plasticity

$$\delta_{pst} = 0.5 - v_o. \tag{11f}$$

This gradient affects strongly the neighboring grains, inducing high tension in them.

## 4. LINKAGE

### 4.1. *Micro vs macro*

The macro-description of the heterogen's degeneration given by eqns (1)–(7) is obtained from the stress-strain curves found in an infinite number of experiments, as can be seen from the references in Blechman (1992). The reality behind this degeneration is the microcracking mechanism which is in turn a result of the macro-action of longitudinal stress. In terms of strains, as given in Part 1 by eqns (15b), (21a) and (25d), the lateral gradient microstrains $\epsilon^*$ are in general the product of the longitudinal macro-deformation $\varepsilon$, realized by the operator $\delta$, which is the gradient factor at microlevel:

$$\epsilon^* = \varepsilon\delta. \tag{12}$$

The weak point of this expression is that it cannot be used to find out the critical macro-values of longitudinal compressive strain $\{\varepsilon_a, \varepsilon_p\}$. There are no difficulties in measuring the critical macrostrains during the test, but to learn why they are critical, we have to check the stress-state of the solid in the opposite direction—from the conditions at microlevel to the macrostate.

### 4.2. *Critical strains*

The strength of solids under load is usually defined as limiting state. Being the peak point of SSc it exists at macrolevel only, referring to the heterogen as a whole. According to Blechman (1992), the limiting state of strength corresponds to the limiting atrophy, $A_p$, when the increment in the energy dissipated through microcracking (degeneration) equals the energy added by loading. The limiting atrophy is the integral (sum) of local micro-atrophies. It is described by eqn (6b) and is reached at the strain $\varepsilon_p$ of the SSc's peak point, given by eqn (9).

It should be underlined that here the macrostate of strength is defined in terms of atrophy, and not in stresses or strains. In contrast to limiting state of strength the term "critical strains" defines in macro the boundaries between the distinct states of the heterogen under load. Correspondingly at the microlevel the microcracking mechanisms are modeled in terms of the strains, related to a single stochastic microcrack. To avoid possible confusion with the term "limiting state" the following description is given in terms of critical strains, at both levels: macro and micro.

4.2.1. *Critical gradient microstrain in general.* The critical gradient strain in micro is that which causes local microrupture. The specificity of this state is that:

(a) due to the locality of the rupture we can take the behavior of the tensioned particle (the acron) as elastic up to appearance of the microcrack;
(b) this rupture is induced by the gradients, not by the full transverse strains;
(c) external lateral compression $\{\sigma_2, \sigma_3\}$ shifts the gradient-induced microrupture to a higher longitudinal strain.

From the above it followed that the gradient critical microstrain equals the resistance of the heterogen to restricted gradient microtension, namely

$$\epsilon^* = \epsilon^R. \tag{13a}$$

Denoting by $\hat{\varepsilon}$ the critical strain of longitudinal compression, which induces the local microrupture, and using eqns (12) and (13a), we find the linkage between the critical micro and macro strains

$$\epsilon^R = \hat{\varepsilon}\delta. \tag{13b}$$

4.2.2. *Critical macrostrain in general.* As a consequence of eqn (13b), the longitudinal critical macrostrain of compression $\hat{\varepsilon}$ is the following function of the local resistance of the heterogen to microrupture $\epsilon^R$ and of the gradient $\delta$:

$$\hat{\varepsilon} = \frac{\epsilon^R}{\delta}. \tag{14}$$

Under multiaxial compression, when $\varepsilon_2 \neq 0$, and the crystalon is fully compressed laterally from the beginning, the gradient strains of tension are realized when Poisson extension exceeds this lateral shortening. The longitudinal strain $\varepsilon_z$, which brings the crystalon back to the state of zero widening, is found from the equation

$$\varepsilon_z \nu_1 - \varepsilon_2 + \varepsilon_3 \nu_3 = 0. \tag{15}$$

For $\varepsilon_3 = \varepsilon_2$ (triaxial compression) and $\nu_2 = \nu_3$ there is

$$\varepsilon_z = \varepsilon_2 \frac{1 - \nu_2}{\nu_1}. \tag{16a}$$

The fraction in (16a) is a magnifier, $M$. Then

$$\varepsilon_z = \varepsilon_2 M. \tag{17}$$

The critical strain for $\varepsilon_2 \neq 0$ will be the sum of (14) and (16b)

$$\hat{\varepsilon} = \frac{\epsilon^R}{\delta} + \varepsilon_2 M. \tag{16b}$$

## 5. STOCHASTICITY

Equation (10) for limiting microrupture, eqns (11) for the gradient factors and eqns (13b), (14), and (16c) for the critical strains are common for brittle solids of different kinds and are expressed in the usual deterministic form. However, two fundamental factors restrict the possibility of using them in this simple way.

First, disorder in the structure of the heterogen, variability in the local features of the components and their combinations, differences in local residual stresses, presence of pores and microflaws and their randomness result in stochasticity in the distribution of local gradients, of local resistance to microrupture and, accordingly in random pattern of microcracks map.

The second factor, Blechman (1988, 1992), is that the limiting atrophy, which is the only cause of heterogen failure, is not linked to the constant value of longitudinal peak strain, but is a function of scattering factor, $d$, and of the atrophy threshold $\varepsilon_a$, given by eqn (6b). These factors should be taken into consideration, when modeling the macro to micro linkage, but the question is a new one, with few experimental data on local resistance to microrupture and on local gradients. Hence, only a preliminary hypothesis can be presented now.

### 5.1. *Levels*

At microlevel, the scattering of the factors in eqns (10) and (11) means that eqn (14) does not represent a single deterministic value, but a *pdf*—a probability density function of the stochastic parameters.

The most promising technique for finding the *pdf* of the microrupture $P(\varepsilon^R)$ is acoustic emission (AE), even if it yields only a part of the *pdf*, being restricted by the peak point of the SSc (appearance of macrocracks suppresses the microcracking process). The positive aspect of the AE-method is that the gradients appear in their true randomness.

Indirectly the $\epsilon^R$-*pdf* can be found from the SSc, when the *pdf* of atrophy given by eqn (7c) is used as its reflection. Thus, the stochasticity of eqn (14), related to the distribution of the limiting gradient strain of microrupture, will in turn bring out the randomness of the gradient factor. (Note: the thrust mechanism is not considered here).

The control parameters of the *pdf* of $P(\varepsilon^R)$ are: $\epsilon^R_{min}$ = the beginning point of this distribution, the probable value of the lower strain limit of the microrupture in gradient tension, and $\epsilon^R_M$ = the mode of this distribution, the strain of maximum likelihood of the *pdf* of heterogen's resistance to microrupture, as described below.

At the macrolevel, besides the modulus of elasticity, only two parameters in the central function are independent and should be linked with the micromechanism: $\varepsilon_a$—the above mentioned atrophy threshold and scattering factor, $d$, or, instead of it, using eqn (8), the mode $\varepsilon_M$ of the *pdf* of the macroatrophy.

Restrictions: at the present state of our test methods and our knowledge of the structure and features of the heterogen's components and their Poisson's ratio, the simplest way to find the macrostrain of the atrophy threshold $\varepsilon_a$ is through macrotests, from the stress-strain relationship.

Then, taking this atrophy threshold from the uniaxial compression test, the lower limit of microrupture can be found from eqn (13b)

$$\epsilon^R_{min} = \varepsilon_a \delta. \tag{18a}$$

Also, the mode, $\epsilon_M^R$ as follows

$$\epsilon_M^R = \varepsilon_M \delta, \tag{18b}$$

where $\varepsilon_M$ is given by eqn (8). However, we still face the problem of measuring the average gradient $\delta$.

5.2. *Two important macrostrains*

Reverting to eqn (14) at macrolevel and using eqn (18), one can relate two important *critical longitudinal* macrostrains: $\varepsilon_a$ and $\varepsilon_M$ in uniaxial compression to their micro counterparts, two critical gradient microstrains:

$$\varepsilon_a = \frac{\epsilon_{min}^R}{\delta}, \tag{19a}$$

$$\varepsilon_M = \frac{\epsilon_M^R}{\delta}. \tag{19b}$$

In the case of triaxial compression (denoted by superscript $T$), the term given by eqn (16b) has to be added. Then

$$\varepsilon_a^T = \frac{\epsilon_{min}^R}{\delta} + \varepsilon_2 M_o, \tag{19c}$$

$$\varepsilon_M^T = \frac{\epsilon_M^R}{\delta} + \varepsilon_2 M_o. \tag{19d}$$

As can be seen from eqns (19), the microcracking process is a result of microresistance to tension (passive element) and of the action of the gradients in the lateral local strains of tension (active element). Appearance of lateral gradients in heterogen under compression is a universal process, and eqns (19) are also universal. Use of (19) for prediction of the features of new heterogens will be possible when new experimental data on $\epsilon_{min}^R$, $\epsilon_M^R$ and $\delta$ are accumulated and generalized.

5.3. *Pdf of microrupture*

As noted above, $\varepsilon_a$ and $\varepsilon_p$ are obtained from compression tests and then the scattering factor is easily found from eqn (3). In general, the macro parameters $\varepsilon_a$ and $d$ fully define the *pdf* of the atrophy of a heterogen in macro.

Now we will try to find the possible *pdf* of the limiting strains of microrupture, $P(\epsilon^R)$, from (7c). By substituting (19c) and (19d) in eqn (8) we find

$$d = \frac{\epsilon_M^R}{\delta} - \frac{\epsilon_{min}^R}{\delta}.$$

The expression

$$d_\epsilon = \epsilon_M^R - \epsilon_{min}^R, \tag{20a}$$

is the scattering factor of local resistance of the heterogen to microrupture. Then

$$d_\epsilon = \delta d. \tag{20b}$$

According to the laws of statistics,

$$P(\epsilon^R) = P_A \frac{\mathrm{d}}{\mathrm{d}\epsilon_R}. \tag{21a}$$

For a rough estimate, taking $\delta$ as constant, eqn (14) yields

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\epsilon_R} = \frac{1}{\delta},$$

and, using expression (7c), we obtain

$$P(\epsilon^R) = \frac{\epsilon^R - \epsilon^R_{min}}{d_\epsilon^2} \exp\left(-0.5 \frac{(\epsilon^R - \epsilon^R_{min})^2}{d_\epsilon^2}\right). \tag{21b}$$

This *pdf* has Rayleigh form and two control parameters: $\{\epsilon^R_{min}, d_\epsilon\}$.

## 6. TENSION-TO-COMPRESSION STRENGTH RATIO FOR CONCRETE

### 6.1. *Value*

The background of the ratio between the strengths of concrete in compression and in tension is an old question, and the gradient models can help us get a possible answer and also verify the model. Due to the similarity between the SSc in compression and tension, the strength formula (4) can be used for both and their ratio, $\theta$, will be

$$\theta = \frac{\sigma_{tens}}{\sigma_{comp}} = \frac{\varepsilon_{pt} E_t G_{pt}}{\varepsilon_{pc} E_c G_{pc}}. \tag{22a}$$

Here, $\sigma_{comp}$, $\sigma_{tens}$ are the strength in compression and tension, respectively, $\varepsilon_{pt}$, $\varepsilon_{pc}$ the strains of SSc peak point for compression and tension, respectively and $p$, $t$, $c$ are the subscripts of the peak point, tension and compression, respectively.

As is well known, $E_t = E_c$. Due to the above mentioned similarity, at the peak point

$$G_{pt} = c_o G_{pc}$$

can be taken. Then

$$\theta = \frac{\varepsilon_{pt}}{\varepsilon_{pc}} c_o. \tag{22b}$$

As seen from Blechman (1988, 1992), the peak strains, $\varepsilon_p$, correspond to specific (absolute) stresses in a live cross-section of a heterogen, and the gradient lateral strains $\varepsilon$ also corresponds to them. Behind the nonlinear domain of SSc for concrete in direct tension there is its own *pdf* of atrophy in the same Rayleigh form, with its own critical parameters, $\varepsilon_{at}$, $\varepsilon_{Mt}$. Naturally, these critical parameters may be not equal to those of gradient microrupture—$\epsilon^R_{min}$, $\epsilon^R_M$. Here we will relate them by a coefficient $c$

$$\varepsilon_{at} = c\epsilon^R_{min}, \tag{23a}$$

$$\varepsilon_{Mt} = c\epsilon^R_M. \tag{23b}$$

Using (8), (20a) and (23) we have

$$d_t = c\epsilon_M^R - c\epsilon_{min}^R = cd_c, \tag{23c}$$

and using eqn (20b) we can write

$$d_c = \frac{d_t}{\delta}. \tag{23d}$$

Substituting the above equations in (9) gives

$$\varepsilon_{pt} = 0.5\varepsilon_{at} + \sqrt{(0.5\varepsilon_{at})^2 + d_t^2} = c(0.5\epsilon_{min}^R + \sqrt{(0.5\epsilon_{min}^R)^2 + d_c^2}). \tag{24a}$$

$$\varepsilon_{pc} = 0.5\varepsilon_{ac} + \sqrt{(0.5\varepsilon_{ac})^2 + d_c^2} = \frac{1}{\delta}(0.5\epsilon_{min}^R + \sqrt{(0.5\epsilon_{min}^R)^2 + d_t^2}). \tag{24b}$$

Finally, substituting the found expressions in (22b) we obtain

$$\theta = c_o c\delta, \tag{25a}$$

or, denoting $C = c_o c$,

$$\theta = C\delta, \tag{25b}$$

indicating that $\theta$ reflects the gradient factor $\delta$.

To check the obtained ratio we can use the data from Soroka (1989), which shows that with increase in concrete strength from 20 MPa, to 60 MPa, $\theta$ decreases from about 0.09 to 0.07, with tensile strength found in the splitting test. For plain concrete with strength 20 MPa, as shown in Part 1, an estimation yields $\delta = 0.06$. Then $C = \theta/\delta = 1.5$. For high-strength concrete with $\delta = 0.40$–$0.045$ the $C$ value is similar.

In the review [Nordijk (1989)] of 78 works, the average ratio between the tensile strength in splitting and in direct tension (p. 37) is 1.14, with the actual values ranging (Table 13.2, p. 95) from 0.88 up to 1.40. Then, the part of $c_o$ may be $1.5/1.14 = 1.32$, which seems improbably high, and therefore we have to conclude that $C$ probably should cover additional factors.

### 6.2. *Strength in compression vs tensile strength*

The fact that $\theta$ reflects the gradient factor can explain the faster increase in the strength of concrete in compression vs its strength in tension. Substituting $C\delta$ for $\theta$ in (22) we have

$$\sigma_{comp} = \frac{\sigma_{tens}}{C\delta}. \tag{26}$$

As a result of (26), when $\delta$ = const and $C$ = const, $\sigma_{comp}$ will be proportional to $\sigma_{tens}$. Yet the increase in both strengths is usually a consequence of parallel improvement in the density and strength of the matrix and its adhesion to the aggregate. With an increase in matrix density its Poisson's ratio also increases and the gradient factor decreases, since the Poisson's ratio of the aggregate does not change. Then the decrease in the denumerator in (26) will add its own contribution to the compressive strength, above the proportionality to the tensile strength.

Note: instead of the compressive strength as reference, the properties of a heterogen in tension are the proper basis for comparisons and estimations.

### 6.3. *Gradient vs tensile strength*

On the evidence of extensive data, [Avram *et al.* (1981), Nordijk (1989), Raphael (1984)] it is common to express the compressive-tensile strength ratio as

$$f'_t = k_1 f_c'^{2/3}. \tag{27a}$$

where $f'_t = \sigma_{tens}$, and $f'_c = \sigma_{comp}$.

However, if the tensile strength is the basis, we have to insert eqn (27a), namely

$$f'_c = k_2(f'_t)^{3/2}. \tag{27b}$$

At the same time, using eqn (26) we can write

$$f'_c = k_3\frac{f'_t}{\delta}. \tag{28}$$

Equaling the above two expressions, we have

$$\frac{1}{\delta} = k_4\sqrt{f'_t}. \tag{29}$$

Since the Poisson gradient is an axially-working parameter, with the strength reflecting the influence of the cross-section, the square root in eqn (29) expresses quantitatively the influence of increasing density of the matrix on the decrease of the Poisson gradient in a matrogen.

It should be noted that eqn (29) derives from eqn (27), which is a regression. This fact restricts the usefulness of eqn (29).

### 6.4. *Splitting tension*

In the splitting tensile test (known as the Brazilian test), the compression is concentrated on a narrow strip, of an area of about 10% from the cross-section of the specimen. The splitting effect is explained in terms of the theory of elasticity by direct tensile stresses induced through the cross-section under the strip (except for small contact zone). The tensile strength is calculated accordingly as

$$\sigma_{spl} = \frac{2F_{spl}}{\pi dl} \tag{30}$$

where $F_{spl}$ is the limiting force of fracture, and $d$, $l$ are the width and length of the specimen, respectively.

Since $\sigma_{spl} = \theta\sigma_{comp}$ we have $F_{spl} = 0.5\pi dl\theta\sigma_{comp}$. Then the compressive stress under the strip, $\sigma_{strip}$ for $\theta = 0.09$, will be

$$\sigma_{strip} = \frac{F_{spl}}{0.1dl} = \frac{\theta\sigma_{comp}\pi dl}{0.2dl} = 1.4\sigma_{comp}.$$

Obviously when $\sigma_{strip}$ exceeds the strength of the specimen in compression by up to 40% (!) there is a very advanced process of gradient microcracking under the strip, and the contribution of the gradient mechanisms in the Brazilian test can drastically change our notion about the cause of the failure in it. As shown by Blechman (1995), lateral tensile rupture under compressive or shear stresses due to gradient-induced confined microtension can take place even in triaxial compression under very high lateral stress.

Note: the average value of $\epsilon^R$, found from the Brazilian splitting test could reflect the real resistance of heterogen to microrupture.

### 6.5. *Distinction*

It is important to recognize the three distinctive types of tension: (a) axial, (b) in splitting and (c) in bending, not as three types of tests but as three modes of the heterogen's behavior under very different strain-stress fields. While axial tension and tension in bending are well recognized by their straightforward effects, there are some problem with a practical aspect of the splitting mechanism. It is not always noted that gradient microtension splits compressed elements and heavily loaded shear zones. It is especially pronounced in rock mechanics, because this type of tension is responsible for the splitting of walls and roofs in mines, rupture of slopes in quarries and so on. It is also involved when earthquakes split the earth's crust (Scholz, 1990).

## 7. SUMMARY

The macrodescription of the nonlinear behavior of a heterogen under compression is given by the central function (1). The Gaussian term in it expresses the probability and the stochastic character of the heterogen's atrophy due to microcracking.

Equations (3) and (8) link four critical parameters: the limiting strain of linearity (atrophy threshold), the peak strain of maximum stress (strength point), the mode of the *pdf* of the atrophy and the scattering factor. These equations enable us to determine any one of them as function of the others. Yet this macrodescription does not include parameters of the microcracking mechanisms.

The micromodels describe the microcracking induced by gradients in Poisson's ratio and in the elastic moduli of the heterogen's components. These gradients induce local lateral tension and rupture in micro.

The strains of longitudinal compression create the lateral gradient strains, but two critical lateral microstrains: $\epsilon^R_{min}$ and $\epsilon^R_M$, affect the two critical macroparameters of the stress-strain relationship: the longitudinal strain of the atrophy threshold $\epsilon_a$, and scattering factor $d$. The linkage between these pairs of critical parameters is probabilistic.

The description of the behavior of a heterogen under load and its strength is obtained in terms of parameters which are commonly measured in tests and have full physical meaning.

To check the presented models the linkage between the strength of concrete in tension and in compression was analyzed, and it was found that their ratio reflects the gradient factor of the heterogen.

As is seen from the gradient mechanisms, the strength of a brittle heterogeneous solid in compression is a function of its resistance to gradient microtension and of the gradient factor. Gradient microrupture can take place even in triaxial compression under very high lateral stresses. When comparing the properties of a heterogen, its features in tension can be a convenient basis.

The fact that the strength ratio reflects the gradient factor, explains the phenomenon of lower increase in the tensile strength of concrete versus its strength in compression, if, as in most cases, the increase in tension strength is accompanied by a decrease of the gradient factor.

The obtained models underscored the distinction between two modes of rupture in tension—one in direct axial tension and the second due to gradient-induced microtension under compression or shear.

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